Maple Worksheets: Getting Started

Tips for getting started:

- Maple commands and variables are case sensitive and order of command execution is important!
- Commands can be executed individually by pressing "Enter" at each command.
- Organize executable commands by placing them after Maple prompts ("[>").
- Use Shift+Enter to create a new line at the same prompt without executing the command.
- Use the "***!!!***" button to execute all commands in your worksheet in order, "***!***" will execute only selected commands.
- You can insert plain text that is not executed as a command, and you can format this text as you would with a word processor.
- You can press Esc or Ctrl+Space to pull up a list of symbols and commands that complete what you just typed.

Starting a New Worksheet: Do This First!

Clearing Variables

At the start of every worksheet, it's a good idea to clear all variables using "restart," otherwise you may get unexpected results!

```
> restart;
```

Loading Additional Packages

You may also want to use some additional Maple packages, load these using "with."

Some tools for creating various plots

```
> with(plots);
```

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, graphplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*] **(1.1)**

Packages useful for Linear Algebra (colon hides output)

```
> with(LinearAlgebra) :
```

Packages for working with Differential Equations

```
> with(DETools) :
```

Basic Command Syntax

Comments

You can create a comment that will not be executed by inserting plain text between your Maple prompts (as I have been doing.) You can also insert a comment using "#"

```
> # 2+2 will be executed, 2+3 will not!
  2 + 2;
   # 2+3;
```

$$4 \tag{2.1}$$

Variables

Values are assigned to variables with "**:=**" not with "=", which is reserved for equations. Notice how Maple handles x and y in the following:

```
> x := a + b;
```

$$x := a + b \tag{2.2}$$

```
> 2·x;
```

$$2\,a + 2\,b \tag{2.3}$$

```
> y = a + b;
```

$$y = a + b \tag{2.4}$$

```
> 2·y;
```

$$2\,y \tag{2.5}$$

This can be useful if we want to assign an equation to a variable name

```
> z := y=a + b;
```

$$z := y = a + b \tag{2.6}$$

```
> 2·z;
```

$$2\,y = 2\,a + 2\,b \tag{2.7}$$

You need to be very careful when defining variables. If you are having problems executing commands in your worksheet, check your variable assignments!

Terminating Statements

Semicolons and colons are used as statement separators in Maple, use these to terminate your statements. Use a semicolon at the end of each statement to display the output once the command is executed. A colon at the end of the statement will execute the command, but will not display the output.

```
> z := a + b :
> z;
```

$$a + b \tag{2.8}$$

Using and Defining Functions and Procedures

Maple has many procedures defined to do various tasks. Procedures will require that you specify certain parameters (some may be optional) which are explained in Maple help, usually along with some examples.

The *solve*(equation(s), variable(s)) procedure takes an equation (or equations) and solves for the specified variable(s). This can also be used to solve inequalities and systems of equations.

```
> solve(a + 2 b + 3 = b^2, b);#solve the quadratic equation for b
```

$$1 + \sqrt{4 + a},\ 1 - \sqrt{4 + a} \tag{3.1}$$

```
> solve( {a + b=3,2 a - 5 b = 2}, [a, b]);#solve the system of equations for a and b
```

$$\left[\left[a = \frac{17}{7},\ b = \frac{4}{7}\right]\right] \tag{3.2}$$

Functions

Mathematical functions are treated as a special type of procedure in Maple, using arrow notation ("->"). For instance, to define $f(x, y) = x^2 + e^y$ as a function:

> $f := (x,y) \to x^2 + e^y;$

$$f := (x, y) \to x^2 + e^y \tag{3.3}$$

Now we can evaluate $f(x, y)$ at $(1, 2)$:

> $f(1, 2);$

$$1 + e^2 \tag{3.4}$$

Procedures

Procedures can extend to more than just mathematical functions, the *solve*( ) procedure used above can take an expression or list as a parameter. Defining our function above as a procedure is slightly more complicated, since we need to specify the input type:

> $g := \mathbf{proc}(x :: numeric, y :: numeric)$
> $x^2 + e^y$
> **end**:

> $g(1, 2);$

$$1 + e^2 \tag{3.5}$$

Palettes and Right Click Context Menus

Palettes

Maple supports many operators, symbols and expressions. The most common of these are available in palettes, so that you can select them without knowing the correct syntax for the procedure and read them in a more familiar format.

The following statements use the Expression palette and the equivalent procedure:

> $\sqrt{4};$
> sqrt(4);

$$2$$
$$2 \tag{4.1}$$

> $\log_{10}(100);$
> log[10](100);

$$2$$
$$2 \tag{4.2}$$

> $\int_0^1 6\,t^2\,\mathrm{d}t;$
> $int(6\,t^2, t = 0\,..1);$

$$2$$
$$2 \tag{4.3}$$

> $\binom{2}{1};$
> binomial(2, 1);

$$2$$
$$2 \tag{4.4}$$

Right Click

Right clicking on an expression will give you a context menu to perform operations without knowing the correct procedure. Notice that some of the following operations result in complex procedures!

Right clicking on the following expression allows us to evaluate for $b = 2$:

> $a + 2b + 3 = b^2$

$$a + 2b + 3 = b^2 \tag{4.5}$$

> $eval(\ a + 2 * b + 3 = b\verb|^|2,\ [b = 2])$

$$a + 7 = 4 \tag{4.6}$$

or solve for b:

> $solve(\ \{\ a + 2 * b + 3 = b\verb|^|2\ \}, [\ b\ ]\ )$

$$\left[\left[b = 1 + \sqrt{4 + a}\right], \left[b = 1 - \sqrt{4 + a}\right]\right] \tag{4.7}$$

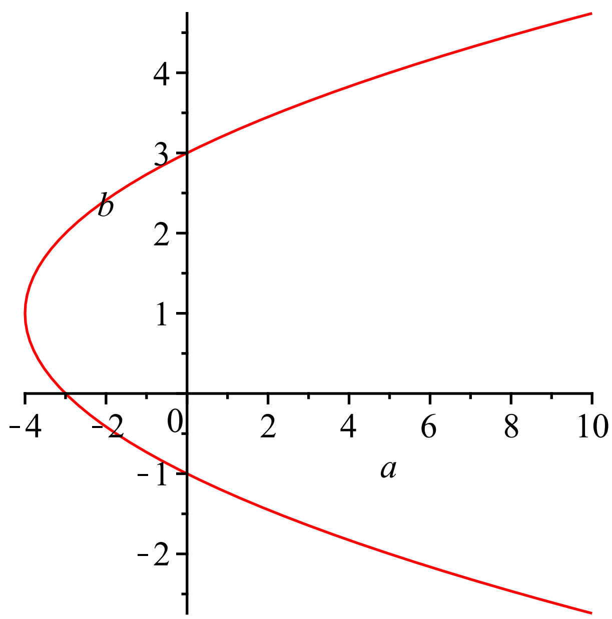
or differentiate implicitly with respect to a:

> $subs(\ b(a) = b, solve(\ diff(subs(\ b = b(a), a + 2 * b + 3 = b\verb|^|2\ ), a), diff(b(a), a)\ )\ )$

$$\frac{1}{2\,(b - 1)} \tag{4.8}$$

or plot:

> $smartplot[a, b](\ a + 2 * b + 3 = b\verb|^|2\ )$



More Examples

From here on, I will assume some familiarity with calculus and linear algebra topics. If something looks unfamiliar, try searching for it in help!

Calculus

We'll work with the following functions for these problems:

> *restart*;

> $f := x \rightarrow 4\,x^3 - 5\,x^2 - x + 6;$
> $g := (x, y) \rightarrow x \cdot e^y + \frac{1}{y};$

$$f := x \rightarrow 4\,x^3 - 5\,x^2 - x + 6$$
$$g := (x, y) \rightarrow x\,e^y + \frac{1}{y} \qquad (5.1)$$

Limits

To evaluate limits we can use the expression palette:

> $\lim_{x \rightarrow \infty} \frac{1}{x};$

$$0 \qquad (5.2)$$

> $\lim_{x \rightarrow 0} \frac{1}{x};$

$$\mathit{undefined} \qquad (5.3)$$

> $\lim_{x \rightarrow 0^-} \frac{1}{x};$

$$-\infty \qquad (5.4)$$

> $\lim_{x \rightarrow 0} \frac{1}{x^2};$

$$\infty \qquad (5.5)$$

> $\lim_{x \rightarrow \infty} \sin(x);$

$$-1\,..1 \qquad (5.6)$$

or we can use the equivalent command *limit*(function, point, direction):

> $\mathit{limit}\left(\frac{1}{x}, x = \mathit{infinity}\right);$

$$0 \qquad (5.7)$$

> $\mathit{limit}\left(\frac{1}{x}, x = 0\right);$

$$\mathit{undefined} \qquad (5.8)$$

> $\mathit{limit}\left(\frac{1}{x}, x = 0, \mathrm{left}\right);$

$$-\infty \qquad (5.9)$$

> $\mathit{limit}\left(\frac{1}{x^2}, x = 0\right);$

$$\infty \qquad (5.10)$$

> $\mathit{limit}(\sin(x), x = \mathit{infinity});$

$$-1\,..1 \qquad (5.11)$$

Differentiation

Maple has several procedures for differentiation, the expression palette contains some familiar notation:

> $\frac{d}{dx} f(x);$

$$12 x^2 - 10 x - 1 \tag{5.12}$$

> $\frac{d^2}{dx^2} f(x);$

$$24 x - 10 \tag{5.13}$$

> $\frac{\partial}{\partial y} g(x, y);$

$$x e^y - \frac{1}{y^2} \tag{5.14}$$

These are equivalent to the *diff*(expression, variables) command:

> $diff(f(x), x);$

$$12 x^2 - 10 x - 1 \tag{5.15}$$

> $diff(f(x), x, x);$

$$24 x - 10 \tag{5.16}$$

> $diff(g(x, y), y);$

$$x e^y - \frac{1}{y^2} \tag{5.17}$$

Another procedure that can be used is the differential operator D[argument indices](function)(evaulation point). This is slightly different from the *diff*() operator, as it can evaluate derivatives at a point, but cannot handle expressions that are not functions:

> $D(f)(x);$

$$12 x^2 - 10 x - 1 \tag{5.18}$$

> $D^{(2)}(f)(x);$

$$24 x - 10 \tag{5.19}$$

> $D(f)(0);$

$$-1 \tag{5.20}$$

> $D[2](g)(x, y);$

$$x e^y - \frac{1}{y^2} \tag{5.21}$$

> $D[1, 2](g)(x, y);$

$$e^y \tag{5.22}$$

> $D[2, 2](g)(x, y);$

$$x e^y + \frac{2}{y^3} \tag{5.23}$$

Integration

Integration is also easily accomplished using the expression palette:

> $\int f(x)\,\mathrm{d}x;$

$$x^4 - \frac{5}{3}x^3 - \frac{1}{2}x^2 + 6x \tag{5.24}$$

> $\int_0^1 f(x)\,\mathrm{d}x;$

$$\frac{29}{6} \tag{5.25}$$

> $\int\int g(x,y)\,\mathrm{d}x\,\mathrm{d}y;$

$$\frac{1}{2}x^2 e^y + x\ln(y) \tag{5.26}$$

Of course there is an equivalent command, *int*(expression, variables):

> $int(f(x),x);$

$$x^4 - \frac{5}{3}x^3 - \frac{1}{2}x^2 + 6x \tag{5.27}$$

> $int(f(x),x=0..1);$

$$\frac{29}{6} \tag{5.28}$$

> $int(g(x,y),x,y);$

$$\frac{1}{2}x^2 e^y + x\ln(y) \tag{5.29}$$

Additional Topics

Maple also has commands for many other specific calculus topics, here are a few that might be useful (look up syntax in the help menu!):

- *taylor* outputs a Taylor series expansion
- *dsolve* is a solver for ordinary differential equations
- *inttrans[laplace]* performs a Laplace transform
- *BesselI, BesselJ, BesselK* and *BesselY* can be used to call the various Bessel functions
- *LegendreP* and *LegendreQ* can be used to call the Legendre functions

Linear Algebra

Remember to load the *LinearAlgebra* package! Notice all of the different commands this will allow us to use, many of which can be very tedious to perform by hand for large matrices:

```
> restart;
> with(LinearAlgebra);
```

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip] **(6.1)**

Matrices

We won't get very far before we need to input a matrix of some sort. We can use the *Matrix()* command to generate some matrices quickly, but it is often simpler to use the Matrix palette. Double click on a large matrix to see and edit its entries:

> $A := Matrix(2, 2, [[1, 2], [2, 4]])$:

> $B := \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$:

> $C1 := \begin{bmatrix} 76 & -61 & 0 & 0 & 0 & 0 \\ -75 & -67 & -50 & 0 & 0 & 0 \\ 0 & 30 & 15 & -89 & 0 & 0 \\ 0 & 0 & -81 & 8 & -18 & 0 \\ 0 & 0 & 0 & -90 & -52 & -58 \\ 0 & 0 & 0 & 0 & 16 & -86 \end{bmatrix}$:

> $C2 := RandomMatrix(100, \text{generator} = 0..9, \text{outputoptions} = [\text{shape} = \text{hermitian}])$;

$$C2 := \begin{bmatrix} \textit{100 x 100 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: triangular}_{\textit{upper}} \\ \textit{Order: Fortran\_order} \end{bmatrix} \qquad \textbf{(6.2)}$$

Vectors

Vectors can be defined using angled brackets (< >):

$> v1 := \langle 1, -1, 1\rangle;$
$v2 := \langle 1, 1, 1\rangle;$
$v3 := \langle 2, 0, 1\rangle;$

$$v1 := \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$v2 := \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$v3 := \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \tag{6.3}$$

Matrix Arithmetic

Matrices can be easily added and subtracted, but there are two kinds of multiplication. We can multiply a matrix by a scalar using *, the product of two matrices is not commutative and uses a period:

$> A + B;$

$$\begin{bmatrix} 6 & 8 \\ 9 & 12 \end{bmatrix} \tag{6.4}$$

$> A - B;$

$$\begin{bmatrix} -4 & -4 \\ -5 & -4 \end{bmatrix} \tag{6.5}$$

$> 5 \cdot A;$

$$\begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix} \tag{6.6}$$

$> A \cdot B;$

```
Error, (in rtable/Product) invalid arguments
```

$> A.B;$

$$\begin{bmatrix} 19 & 22 \\ 38 & 44 \end{bmatrix} \tag{6.7}$$

$> B.A;$

$$\begin{bmatrix} 17 & 34 \\ 23 & 46 \end{bmatrix} \tag{6.8}$$

Useful Functions

We can use commands to manipulate matrices quickly (see the output of *with(LinearAlgebra)* above). Here are a few examples:

> $Determinant(A);$

$$0 \tag{6.9}$$

> $Eigenvalues(A);$

$$\begin{bmatrix} 0 \\ 5 \end{bmatrix} \tag{6.10}$$

> $Eigenvectors(A);$

$$\begin{bmatrix} 5 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} & -2 \\ 1 & 1 \end{bmatrix} \tag{6.11}$$

> $LUDecomposition(A);$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \tag{6.12}$$

> $QRDecomposition(A);$

$$\begin{bmatrix} \frac{1}{5}\sqrt{5} \\ \frac{2}{5}\sqrt{5} \end{bmatrix}, \begin{bmatrix} \sqrt{5} & 2\sqrt{5} \end{bmatrix} \tag{6.13}$$

> $GramSchmidt(\{\mathrm{v1}, \mathrm{v2}, \mathrm{v3}\});$

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{2}{3} \\ \frac{4}{3} \\ \frac{2}{3} \end{bmatrix}, \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{bmatrix} \right\} \tag{6.14}$$

> $GramSchmidt(\{\mathrm{v1}, \mathrm{v2}, \mathrm{v3}\}, normalized);$

$$\left\{ \left[\begin{bmatrix} \frac{1}{6}\sqrt{6} \\ \frac{1}{3}\sqrt{6} \\ \frac{1}{6}\sqrt{6} \end{bmatrix}, \begin{bmatrix} \frac{1}{2}\sqrt{2} \\ 0 \\ -\frac{1}{2}\sqrt{2} \end{bmatrix}, \begin{bmatrix} \frac{1}{3}\sqrt{3} \\ -\frac{1}{3}\sqrt{3} \\ \frac{1}{3}\sqrt{3} \end{bmatrix} \right] \right\} \tag{6.15}$$